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RESPONSE OF A STRAND WITH ELLIPTICAL OUTER WIRES

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Abstract—The linearized form of the equilibrium equations for wire rope is used to develop a theory which describes the axial response of a strand, with elliptical outer wire cross-sections, to a static load for two strand configurations. The strand geometries are the same except for a small flat surface on each of the outer wires for the second configuration. The forces, moments, stresses and strains are calculated using the same loading conditions for each strand configuration and varying the ellipticity of the outer wires. The results for both cases are then evaluated and an application of the theory is briefly discussed.

NOMENCLATURE

a_2	major axis of outer wire
b_2	minor axis of outer wire
d	original outer diameter
m_2	number of wires
r_2	initial helix radius
E	modulus of elasticity
F_1	axial force on center wire
F_2	axial force on outer wires
F_T	total axial load on wire rope
G_2	bending moment in an outer wire
H_2	axial twisting moment in an outer wire
M_1	axial twisting moment on center wire
M_2	axial twisting moment on outer wires
M_T	total moment on wire rope
N_2	shearing force in an outer wire
R_1	radius of center wire
S	shearing stress in an outer wire
T_2	axial force in an outer wire
X_2	contact force between an outer wire and center wire
α_2	initial helix angle of wire
$\bar{\alpha}_2$	final helix angle of wire
$\Delta\alpha_2$	change in helix angle
β_2	rotational strain of an outer wire
γ	ratio of minor axis to major axis in ellipse
$\Delta\kappa_2$	change in curvature of an outer wire
ν	Poisson's ratio
ρ_1	radius of curvature of center wire
ρ_2	radius of curvature center
σ_1	axial stress in center wire
σ_N	axial stress in an outer wire
σ_B	bending stress in an outer wire
σ_C	contact stress between the center wire and an outer wire
τ	angle of twist per unit length of strand
$\Delta\tau_2$	change in twist per unit length of an outer wire
ξ_1	axial strain in center wire
ξ_2	axial strain in outer wire.

INTRODUCTION

In previous research, Costello and Velinsky (1980) used general non-linear equilibrium equations to describe the axial response of a strand with oval outer wires. These equations were develolped by separating the rope into thin helical wires and applying a solution of the general non-linear set of equations for the twisting and bending of a thin rod. This work

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was based on a geometry of m smooth rods whose shapes in the unstressed state were helical with elliptical cross-sections and no center wire.

More recently, Costello (1983) and Velinsky *et al.* (1984) have developed a set of linearized equilibrium equations to describe the static response of a strand and a wire rope. These equations are an extension of the frictionless theory to multilayer cables and are applied to a wire rope with a center wire and m outer wires, all having circular cross-sections.

In this paper, the linearized form of the equilibrium equations is used to describe the axial response of two different strand configurations with no end rotation. The first strand configuration consists of a center wire having a circular cross-section and six helical outer wires with elliptical cross-sections that only make contact with the center wire. The axial, bending, shearing and contact stresses are shown to decrease for the same axial load as the outer wires become more elliptical (increase in eccentricity) while the original outer diameter of the strand is kept constant.

The second strand configuration consists of a center wire having a circular cross-section and six helical outer wires each with a circular cross-section, except for a small flat surface which only makes contact with the center wire. The contact stresses in this configuration are shown to decrease significantly compared to a configuration with no flat surface on each of the outer wires. The minimal effects of wire curvature were not considered when calculating these contact stresses.

ANALYSIS

In order to demonstrate the reduction of stresses caused by a tensile axial load, the linearized form of the equilibrium equations developed by Costello and others are used (Costello, 1983; Phillips *et al.*, 1980; Velinsky and Costello, 1980). Some of these equations have been modified to include elliptical wire cross-sectional dimensions.

For the purpose of this paper, a strand has a center wire of radius R_1, and m_2 helical outer wires with cross-sections having a major axis radius a_2 and a minor axis radius b_2, such that

$$b_2 = \frac{\gamma \sin \alpha_2 d}{2[\gamma \sin \alpha_2 + [\gamma^2 \sin^2 \alpha_2 + \tan^2((\pi/2) - (\pi/m_2))]^{0.5}]} \tag{1}$$

where d is the original outer diameter of wire rope, and $\gamma = b_2/a_2$. This allows the outer wires of the unloaded strand to just come in contact with each other. In actual practice, b_2 would be slightly smaller and R_1 would be slightly larger than the calculated values for b_2 and R_1 in order to prevent this contact. This small change in b_2 and R_1 does not significantly affect the following results.

The initial helix radius r_2 is

$$r_2 = R_1 + b_2 = b_2\left[1 + \frac{\tan^2((\pi/2) - (\pi/m_2))}{\gamma^2 \sin^2 \alpha^2}\right]^{0.5} \tag{2}$$

and the initial helix angle α_2 is

$$\alpha_2 = \tan^{-1} \frac{p_2}{2\pi r_2} \tag{3}$$

where p_2 is the initial pitch of the outside wires.

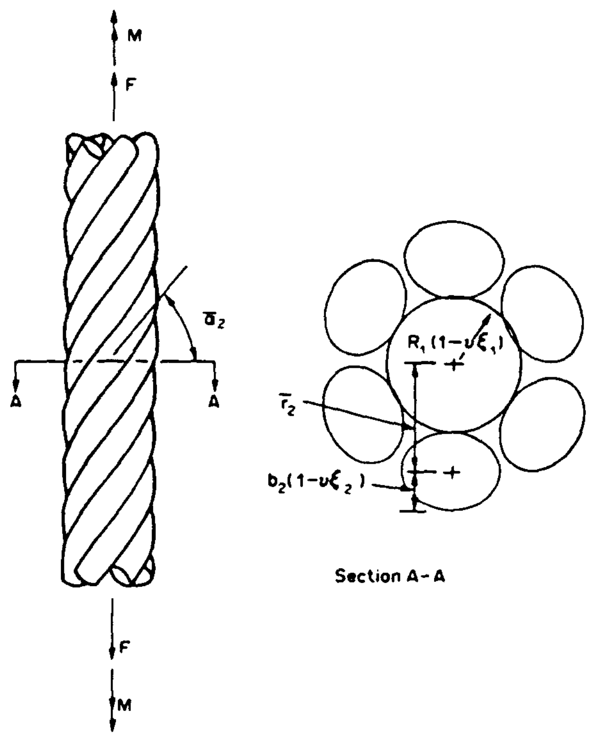

Fig. 1. Geometry of a simple strand.

When the wire rope is loaded (see Fig. 1), the center wire assumes a new wire radius, $R_1(1-\nu\xi_1)$, and the outer wire assumes a new minor axis, $b_2(1-\nu\xi_2)$, where ξ_1 is the axial strain in the center wire, and ξ_2 is the axial strain in an outer wire and ν is Poisson's ratio. The outer wires, remaining helical, have a new helix angle, $\bar{\alpha}_2$. The increase in helix angle, $\Delta\alpha_2 = \bar{\alpha}_2 - \alpha_2$, can be expressed as

$$\Delta\alpha_2 = \frac{r_2 \tan\alpha_2[\xi_1(1+\nu) - \beta_2 \tan\alpha_2]}{r_2 \tan^2\alpha_2 + r_2 + \nu b_2} \tag{4}$$

where β_2 is the rotation of an outer wire defined as $\beta_2 = r_2\tau$, where τ is the angle of twist per unit length of the strand.

The axial strain ξ_1 in the center wire is

$$\xi_1 = \frac{F_T - m_2[T_2 \sin\alpha_2 + N_2 \cos\alpha_2]}{\pi E R_1^2} \tag{5}$$

where F_T is the total axial load, N_2 is the shearing force in an outer wire and T_2 is the axial force in an outer wire. As can be seen from the results in Table 3, $N_2 \ll T_2$, therefore the contribution of the shear force in eqn (5) can be neglected.

The axial strain, in the center wire is thus

$$\xi_1 = \frac{F_T[r_2(\tan^2\alpha_2 + 1) + \nu b_2] - m_2\pi E a_2 b_2 r_2 \beta_2 \sin\alpha_2 \tan\alpha_2}{\pi E[(R_1^2 + m_2 a_2 b_2 \sin\alpha_2)(r_2 \tan^2\alpha_2 + r_2 + \nu b_2) - m_2 a_2 b_2 r_2(1+\nu)\sin\alpha_2]}. \tag{6}$$

The axial strain ξ_2 in the outer wires can then be shown to be

$$\xi_2 = \xi_1 - \frac{\Delta\alpha_2}{\tan\alpha_2}. \tag{7}$$

The change in curvature $\Delta\kappa_2$ for the outer wires is

$$\Delta\kappa_2 = -\frac{2\sin\alpha_2\cos\alpha_2}{r_2}\Delta\alpha_2 + \frac{\nu\cos^2\alpha_2}{r_2^2}(R_1\xi_1 + b_2\xi_2). \tag{8}$$

The angle of twist per unit length $\Delta\tau_2$ is

$$\Delta\tau_2 = \frac{1-2\sin^2\alpha_2}{r_2}\Delta\alpha_2 + \frac{\nu\sin\alpha_2\cos\alpha_2}{r_2^2}(R_1\xi_1 + b_2\xi_2). \tag{9}$$

Listed below are the force and moment equations for the loaded outer wires in static equilibrium (see Fig. 3):

$$G_2 = \frac{\pi}{4}Ea_2b_2^3\Delta\kappa_2 \tag{10}$$

$$H_2 = \frac{\pi Ea_2^3b_2^3}{2(1+\nu)(a_2^2+b_2^2)}\Delta\tau_2 \tag{11}$$

$$N_2 = H_2\frac{\cos^2\alpha_2}{r_2} - G_2\frac{\sin\alpha_2\cos\alpha_2}{r_2} \tag{12}$$

$$T_2 = \pi Ea_2b_2\xi_2 \tag{13}$$

$$X_2 = N_2\frac{\sin\alpha_2\cos\alpha_2}{r_2} - T_2\frac{\cos^2\alpha_2}{r_2} \tag{14}$$

$$F_2 = m_2[T_2\sin\alpha_2 + N_2\cos\alpha_2] \tag{15}$$

and

$$M_2 = m_2[H_2\sin\alpha_2 + G_2\cos\alpha_2 + T_2r_2\cos\alpha_2 - N_2r_2\sin\alpha_2] \tag{16}$$

where E is the modulus of elasticity of the wire material; G_2 is the bending moment in an outer wire; H_2 is the axial twisting moment in an outer wire; N_2 is the shearing force in an outer wire; T_2 is the axial force in an outer wire; X_2 is the resultant contact force per unit length acting on an outer wire; F_2 is the total axial force acting on the outer wires; and M_2 is the total axial twisting moment acting on the outer wires.

For the center wire

$$F_1 = \pi ER_1^2\xi_1 \tag{17}$$

and

$$M_1 = \frac{\pi ER_1^2\beta_2}{4(1+\nu)r_2} \tag{18}$$

where F_1 is the axial force acting on the center wire, and M_1 is the axial twisting moment acting on the center wire.

The total force (F_T) and total moment (M_T) acting on the wire rope are

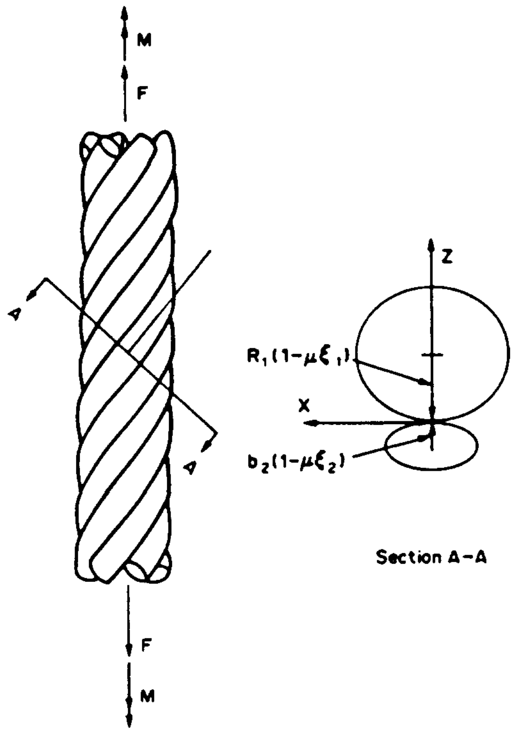

Fig. 2. View of a plane section perpendicular to an outer wire.

$$F_T = F_1 + F_2 \tag{19}$$

and

$$M_T = M_1 + M_2. \tag{20}$$

Using these static equilibrium equations, the resultant stresses are

$$\sigma_A = \frac{T_2}{\pi a_2 b_2} \tag{21}$$

$$\sigma_B = \frac{4G_2}{\pi a_2 b_2^2} \tag{22}$$

$$S = \frac{2H_2}{\pi a_2 b_2^2} \tag{23}$$

$$\sigma_1 = \frac{F_1}{\pi R_1^2} \tag{24}$$

where σ_A is the axial stress in an outer wire; σ_B is the bending stress in an outer wire; S is the shearing stress in an outer wire; and σ_1 is the axial stress in center wire.

In calculating the contact stress (Boresi and Sidebottom, 1985) between an outer wire and the center wire (see Fig. 2), it is assumed that the contact surface is along a straight

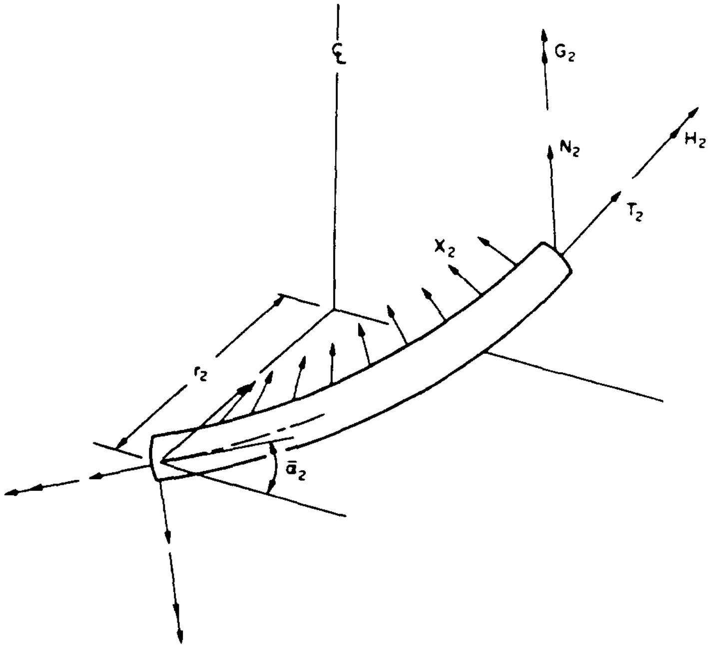

Fig. 3. Forces and moments acting on a loaded helical wire in equilibrium.

line element. Also, the strand is sufficiently long to disregard the radii of curvature which lie in the plane of the line of contact. Thus, only the radii of curvature perpendicular to the line of contact are used to calculate the contact stress. The contract stress can then be expressed as

$$\sigma_c = -\frac{b}{\Delta} \tag{25}$$

where

$$b = (-2X_2\Delta/\pi)^{0.5} \tag{26}$$

and

$$\frac{2[(1-\nu^2)/E]}{\dfrac{1}{2\rho_1}+\dfrac{1}{2\rho_2}}. \tag{27}$$

The radii of curvature for the first strand configuration, ρ_1 and ρ_2 are

$$\rho_1 = \frac{R_1}{\sin^2\alpha_2} \tag{28}$$

$$\rho_2 = \frac{b}{\gamma^2}. \tag{29}$$

For the second wire rope configuration which includes a "flat spot" on the outer wires, the radius of curvature ρ_1 is the same as in eqn (28) and $\rho_2 = \infty$, thus reducing eqn (27) to

$$\Delta = 4\rho_1[(1-\nu^2)/E]. \tag{30}$$

Table 1

Modulus of elasticity (E)	3.00E+07 psi
Poisson's ratio (ν)	0.25
Maximum outer diameter of cable (d)	0.500 inches
Number of wires in second layer (m_2)	6
Total axial load (F_T)	10,000 pounds
Rotational rope strain (β_2)	0 radians
original helix angle of wire (α_2)	75 degrees

RESULTS

As stated in the Introduction, the first strand configuration consists of a center wire with a circular cross-section and six helical outer wires with elliptical cross-sections that only make contact with the center wire. The second strand configuration is the same except for a small flat surface on each of the outer wires which only makes contact with the center wire. The parameters that are kept constant for both configurations are shown in Table 1. The modulus of elasticity E, and Poisson's ratio ν, are properties of the material used in the wire rope. The original outer diameter d, number of wires m_2 and the rotational strain of an outer wire β_2 are the geometrical constraints. All of the internal forces, loads and strains are expressed in terms of the two independent variables, the total axial load on wire rope F_T and the initial helix angle of the outer wire α_2.

Table 2 shows the resulting forces, moments, stresses and strains from eqns 1, 2, 6, 17, 18 and 24 for the core wire of a steel strand under a 10,000 pound load and no end rotation. Each column of data is based on the geometry of an outer wire in which the minor axis radius b_2 is reduced by 5% of the major axis radius a_2. In order to maintain a constant outer diameter d, the core wire radius R_1 must increase as the minor axis b_2 decreases. This accounts for the increase in the axial force F_1 and the reduced strain as the outer wires become more eccentric. The moment on the core wire is zero because the strand is prevented from having an end rotation β_2.

The forces, moments, stresses and strains from eqns 1, 2, 4, 7–16, 21–23 and 24 for the outer wires with the same geometric constraints as the core wire are shown in Table 3. The load-carrying capacity of the outer wires decreases with increased eccentricity of each outer wire because of the reduction in the cross-sectional areas. The axial, σ_A, bending, σ_B and contact stresses, σ_{C1} and σ_{C2}, reduce linearly (see Figs 4, 5, 7), while the shearing stress S reduces non-linearly (see Fig. 6) because of the reduction of the axial load F_2. The strand geometry which includes the "flat spot" reduces the contact stress between an outer wire and the core wire from 257,570 to 176,536 psi for the concentric outer wire, a reduction of over 30%. The contact stress between the most eccentric outer wire and the core wire is reduced over 26% by including the "flat spot" on the outer wire. The change in the reduction of the contact stress is due to the increased radius of curvature at the point of contact.

Table 4 lists the total axial load F_T, strain ξ_1 and moment M_T on the strand from eqns 6, 17 and 18. The slight increase in the axial load F_T from the given value in Table 1 is due to the contribution of the shearing force in the outer wires. This total shearing force is negligible in the case of concentric outer wires and reduced with eccentricity. This is why the contribution of the shearing force in an outer wire N_2 (1/6 of the total shearing force) is neglected when calculating the axial strain, in the center wire, ξ_1.

CONCLUSIONS

An investigation of the axial response for two strand configurations has been performed.

The geometric constraints for both configurations include:

(a) a constant outer diameter d
(b) no end rotation ($\beta = 0$)
(c) the outer wires only make contact with the center wire.

Table 2

Core wire	$A = B$	$0.95A = B$	$0.90A = B$	$0.85A = B$	$0.80A = B$	$0.75A = B$	$0.70A = B$	$0.65A = B$	$0.60A = B$
Wire radius (R_1)	0.08623	0.09056	0.09517	0.10006	0.10526	0.11079	0.11666	0.12291	0.12956
Axial force (F_1)	1726.69	1874.73	2037.77	2217.42	2415.45	2633.82	2874.63	3140.17	3432.90
Axial strain in core (ξ_1)	0.00246	0.00243	0.00239	0.00235	0.00231	0.00228	0.00224	0.00221	0.00217
Axial stress (σ_1)	73910	72757	71621	70502	69398	68308	67230	66161	65098
Total moment (M_1)	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

Table 3

Second layer of wires	$A = B$	$0.95A = B$	$0.90A = B$	$0.85A = B$	$0.80A = B$	$0.75A = B$	$0.70A = B$	$0.65A = B$	$0.60A = B$
Helix radius of wires r_2	0.16812	0.17028	0.17258	0.17503	0.17763	0.18039	0.18333	0.18646	0.18978
Minor axis radius b_2	0.08188	0.07972	0.07742	0.07497	0.07237	0.06961	0.06667	0.06354	0.06022
Major axis radius a_2	0.08188	0.08391	0.08602	0.08820	0.09046	0.09281	0.09524	0.09776	0.10037
Change in helix angle $\Delta\alpha_2$	0.00076	0.00075	0.00074	0.00073	0.00072	0.00071	0.00070	0.00069	0.00067
Axial strain in wire ξ_2	0.00226	0.00222	0.00219	0.00215	0.00212	0.00209	0.00205	0.00202	0.00199
Change of curvature $\Delta\kappa_2$	−0.00204	−0.00198	−0.00192	−0.00187	−0.00181	−0.00175	−0.00170	−0.00165	−0.00159
Change in angle of twist $\Delta\tau_2$	−0.00306	−0.00297	−0.00288	−0.00280	−0.00271	−0.00263	−0.00255	−0.00246	−0.00238
Bending moment G_2	−2.15630	−1.98207	−1.80774	−1.63426	−1.46272	−1.29430	−1.13033	−0.97223	−0.82150
Bending stress σ_B	−5000.81	−4732.45	−4464.55	−4197.22	−3930.57	−3664.72	−3399.83	−3136.10	−2873.74
Twisting moment H_2	−2.58877	−2.50097	−2.39701	−2.27652	−2.13952	−1.98657	−1.81884	−1.63882	−1.44744
Shearing stress S	−3001.88	−2985.70	−2959.95	−2923.36	−2874.63	−2812.42	−2735.37	−2642.18	−2531.69
Axial forces in wire T_2	1427.53	1401.98	1373.85	1342.85	1308.68	1271.01	1229.45	1183.64	1133.13
Shearing force N_2	2.17504	1.92612	1.68826	1.46300	1.25182	1.05603	0.87679	0.71500	0.57126
Contact force S_2	−565.57	−548.70	−530.81	−511.85	−491.77	−470.51	−448.03	−424.28	−399.21
Contact stress (1) σ_{C1}	−257570	−245818	−234071	−222329	−210593	−198862	−187135	−175414	−163695
Contact stress (2) σ_{C2}	−176536	−169675	−162801	−155911	−148999	−142061	−135088	−128072	−121002
Axial force second layer F_2	8276.68	8128.26	7964.85	7784.86	7586.49	7367.82	7126.73	6860.94	6567.99
Total moment M_2	352.215	351.258	349.813	347.778	345.030	341.425	336.796	330.947	323.654

Table 4

Total values	$A = B$	$0.95A = B$	$0.90A = B$	$0.85A = B$	$0.80A = B$	$0.75A = B$	$0.70A = B$	$0.65A = B$	$0.60A = B$
Total axial load F_T	10003.38	10002.99	10002.62	10002.27	10001.94	10001.64	10001.36	10001.11	10000.89
Total axial strain ξ_1	0.00246	0.00243	0.00239	0.00235	0.00231	0.00228	0.00224	0.00221	0.00217
Total moment M_T	352.22	351.26	349.81	347.78	345.03	341.42	336.80	330.95	323.65

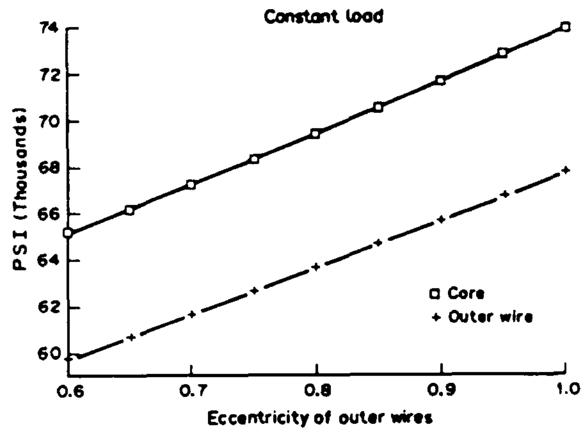

Fig. 4. Axial stresses in strand.

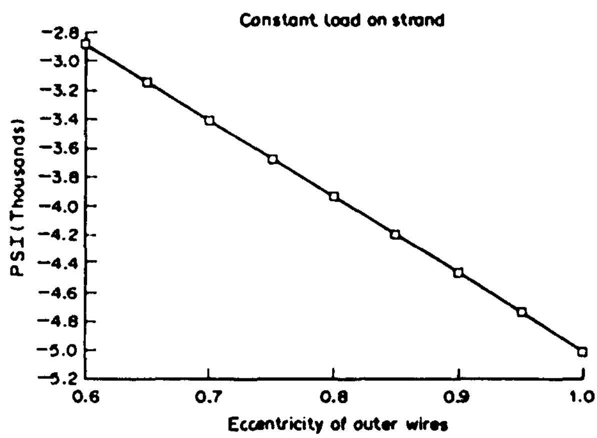

Fig. 5. Bending stress in an outer wire.

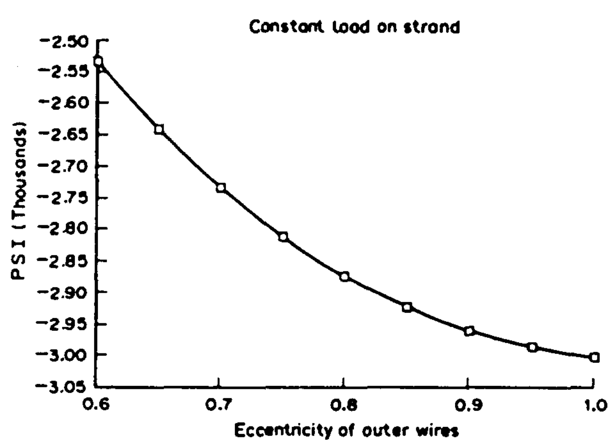

Fig. 6. Shearing stress in an outer wire.

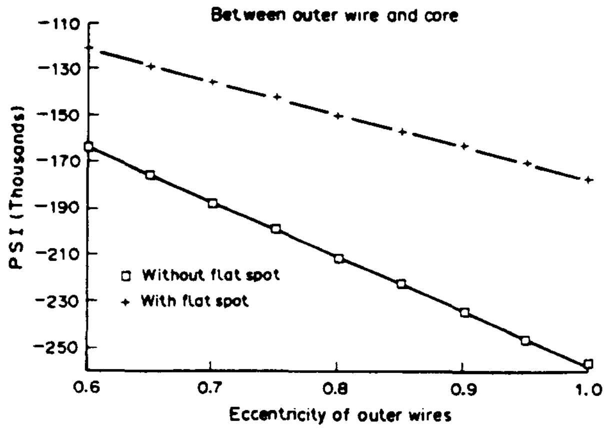

Fig. 7. Contact stress.

For the first configuration consisting of a center wire having a circular cross-section and six helical outer wires with elliptical cross-sections the following conclusions can be made.

(1) With the total axial load F_T and the original helix angle α_2 held constant the percentage of the total load that the center wire supports increases and the overall strains and moments in the rope decrease because of the change in the center wire radius.

(2) The calculated internal stresses are reduced as the outer wires increase their ellipticity.

Since the second configuration is the same as the first except for the small flat surface on each of the outer wires, the same conclusions as listed above apply, as well as:

(1) compared with the first configuration, the flat surface significantly reduces the contact stress between the outer wires and the center wire;

(2) this difference decreases as the outer wires become more eccentric.

If different materials are available for strand construction, the theory can be used to optimize the design of the strand. For example, if a constant diameter, pitch of the outer wires and number of wires in the second layer are the geometric constraints for the strand design and the maximum load is known, the design can be modified for each of the materials. In this way, a less costly material may be used to support the same given load. By using this theory, other parameters can be modified to optimize the design of a strand.

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